Journal of Applied Mechanics and Technical Physics, Vol. 42, No. 3, pp. 510–515, 2001

VISCOELASTIC VIBRATIONS OF A TRIANGULAR PLATE

N. A. Chernyshov and A. D. Chernyshov

UDC 534.121/122

The problem of stationary vibrations of a viscoelastic equilateral triangular plate is treated. The vibrations are caused by the action of a uniformly distributed load which varies harmonically and by the motion of the simply supported boundary of the plate as a rigid body which vibrates at the same frequency. The level lines of the vibration amplitude are studied and the graphs of the amplitude distribution along the height of the triangle are given.

Sing, Ahmad, and Hilton [1] performed a finite-element analysis of vibrations of an elastic plate covered by a viscoelastic damping layer and studied the effect of the parameters of the viscoelastic layer, the temperature, and the elastic moduli of the plate on damping of free vibrations. Yuanyuan an Changjun [2] studied analytically the stability of simply-supported, compressed viscoelastic rectangular plates on a viscoelastic foundation, developed a model of the stability problem, and derived differential equations that govern the viscoelastic deformation of the plate. Boshnich [3] proposed a method of calculating the forced transverse vibrations of thin rectangular plates with allowance for energy losses in a cyclically deformed material upon forced harmonic excitation. In [3], the vibrations of a nonconservative elastic plate system were studied by asymptotic methods of the nonlinear mechanics. Chernyshov and Chernyshov [4] obtained an exact solution of the problem of vibrations of a triangular plate and gave expressions for resonance frequencies.

1. Formulation of the Problem. For the linear stress state of a viscoelastic body, the stress tensor σ_{ij} is expressed in terms of the strain tensor e_{ij} and the strain-rate tensor ε_{ij} [5]:

$$\sigma_x = \lambda_e(e_x + e_y) + 2\mu_e e_x + \lambda_\nu(\varepsilon_x + \varepsilon_y) + 2\mu_\nu \varepsilon_x,$$

$$\sigma_y = \lambda_e(e_x + e_y) + 2\mu_e e_y + \lambda_\nu(\varepsilon_x + \varepsilon_y) + 2\mu_\nu \varepsilon_y, \qquad \tau_{xy} = 2\mu_e e_{xy} + 2\mu_\nu \varepsilon_{xy}.$$

Here λ_e and μ_e are the Lamé coefficients and λ_ν and μ_ν are the viscosity coefficients. According to [5], these formulas can be rewritten in the form

$$\sigma_x = \frac{E}{1-\nu^2}(e_x + \nu e_y) + \frac{E_\nu}{1-\nu_\nu^2}(\varepsilon_x + \nu_\nu \varepsilon_y),$$

$$\sigma_y = \frac{E}{1-\nu^2}(e_y + \nu e_x) + \frac{E_\nu}{1-\nu_\nu^2}(\varepsilon_y + \nu_\nu \varepsilon_x), \qquad \tau_{xy} = G e_{xy} + G_\nu \varepsilon_{xy}.$$

Here

$$\lambda_e + 2\mu_e = \frac{E}{1-\nu^2}, \qquad \lambda_e = \frac{E\nu}{1-\nu^2}, \qquad G = 2\mu_e,$$
$$\lambda_\nu + 2\mu_\nu = \frac{E_\nu}{1-\nu_\nu^2}, \qquad \lambda_\nu = \frac{E_\nu \nu_\nu}{1-\nu_\nu^2}, \qquad G_\nu = 2\mu_\nu. \tag{1.1}$$

The use of similar expressions for the viscous and elastic constants in Eq. (1.1) makes it possible to write the dynamic equation of motion of a plate relative to the deflection W in a convenient form

$$D_e \nabla^4 W + D_\nu \nabla^4 W_t = q - \rho H W_{tt}, \quad D_e = \frac{EH^3}{12(1-\nu^2)}, \quad D_\nu = \frac{E_\nu H^3}{12(1-\nu_\nu^2)}. \tag{1.2}$$

Voronezh State Technology Academy, Voronezh 394000. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 42, No. 3, pp. 152–158, May–June, 2001. Original article submitted July 13, 2000; revision submitted November 9, 2000.

 0021-8944/01/4203-0510 $25.00

Let a viscoelastic equilateral triangular plate be subjected to the action of a uniformly distributed load which varies according to the law $q = q_0 \cos\omega t$. The boundary of the simply supported plate vibrates harmonically as a rigid body:

$$W\Big|_\Gamma = a\cos\omega t + b\sin\omega t, \qquad \frac{\partial^2 W}{\partial n^2}\Big|_\Gamma = 0. \tag{1.3}$$

We seek the solution of Eq. (1.2) in the form

$$W = U(x,y)\cos\omega t + V(x,y)\sin\omega t. \tag{1.4}$$

Substituting W from (1.4) into Eq. (1.2), we obtain the following system of differential equations for U and V:

$$D_e\nabla^4 U + D_\nu\omega\nabla^4 V = q_0 + \rho H\omega^2 U, \qquad D_e\nabla^4 V - D_\nu\omega\nabla^4 U = \rho H\omega^2 V \tag{1.5}$$

(∇^2 is the Laplace operator).

To eliminate $\nabla^4 V$ and V from the second differential equation of (1.5), we express $\nabla^4 V$ from the first equation:

$$\nabla^4 V = \frac{q_0 + \rho H\omega^2 U - D_e\nabla^4 U}{D_\nu\omega}. \tag{1.6}$$

Elimination of $\nabla^4 V$ from the second equation (1.5) yields

$$D_\nu^2\omega^2\nabla^4 U = -D_\nu\rho H\omega^3 V + D_e(q_0 + \rho H\omega^2 U - D_e\nabla^4 U). \tag{1.7}$$

We apply the operator ∇^4 to the left and right sides of Eq. (1.7) and use (1.6) to eliminate $\nabla^4 V$ from the resulting equation. As a result, we obtain the eighth-order differential equation for the unknown function U:

$$(D_e^2 + D_\nu^2\omega^2)\nabla^8 U - 2\rho H\omega^2 D_e\nabla^4 U + (\rho H\omega^2)^2 U = -\rho H\omega^2 q_0. \tag{1.8}$$

We find the function V from (1.7):

$$V = \frac{\rho H\omega^2 D_e U - (D_e^2 + D_\nu^2\omega^2)\nabla^4 U + D_e q_0}{\rho H\omega^3 D_\nu}. \tag{1.9}$$

Substituting (1.4) into (1.3), we obtain the following boundary conditions for U and V:

$$U\Big|_\Gamma = a, \qquad \frac{\partial^2 U}{\partial \boldsymbol{n}^2}\Big|_\Gamma = 0, \qquad V\Big|_\Gamma = b, \qquad \frac{\partial^2 V}{\partial \boldsymbol{n}^2}\Big|_\Gamma = 0. \tag{1.10}$$

The differential equation (1.8) subject to four boundary conditions (1.10) is of the eighth order. Because of the high order, serious difficulties arise in the construction of an exact solution of the above-formulated problem.

2. Construction of the Exact Solution. To find a particular solution of the inhomogeneous differential equation (1.8) for U, we assume that the function U depends only on one geometrical coordinate x, i.e., $U = U(x)$. We call the function $U(x)$ the fundamental function and denote it by $F(x)$. From (1.8), we obtain the following ordinary linear differential equation for $F(x)$:

$$(D_e^2 + D_\nu^2\omega^2)F^{\mathrm{VIII}} - 2\rho H\omega^2 D_e F^{\mathrm{IV}} + (\rho H\omega^2)^2 F = -\rho H\omega^2 q_0. \tag{2.1}$$

We find the general solution of the homogeneous equation

$$(D_e^2 + D_\nu^2\omega^2)F^{\mathrm{VIII}} - 2\rho H\omega^2 D_e F^{\mathrm{IV}} + (\rho H\omega^2)^2 F = 0.$$

Its characteristic equation $(D_e^2 + D_\nu^2\omega^2)\alpha^{\mathrm{VIII}} - 2\rho H\omega^2 D_e\alpha^{\mathrm{IV}} + (\rho H\omega^2)^2 = 0$ has the following roots:

$$\alpha_{1,2,3,4} = \pm\sqrt{\frac{\sqrt{\rho H\omega^2}\Big(\pm\sqrt{D_e + \sqrt{D_e^2 + D_\nu^2\omega^2}} + \sqrt[4]{4D_e^2 + 4D_\nu^2\omega^2}\Big)}{\sqrt{8D_e^2 + 8D_\nu^2\omega^2}}},$$

$$\alpha_{5,6,7,8} = \pm i\sqrt{\frac{\sqrt{\rho H\omega^2}\Big(\pm\sqrt{D_e + \sqrt{D_e^2 + D_\nu^2\omega^2}} + \sqrt[4]{4D_e^2 + 4D_\nu^2\omega^2}\Big)}{\sqrt{8D_e^2 + 8D_\nu^2\omega^2}}}.$$

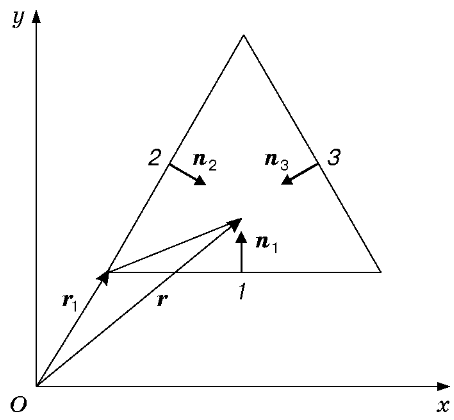

Fig. 1

The solution of the inhomogeneous equation (2.1) is given by

$$F(x) = \mathrm{e}^{\lambda x}(C_1 \cos æx + C_2 \sin æx) + \mathrm{e}^{-\lambda x}(C_3 \cos æx + C_4 \sin æx)$$
$$+ \mathrm{e}^{æx}(C_5 \cos \lambda x + C_6 \sin \lambda x) + \mathrm{e}^{-æx}(C_7 \cos \lambda x + C_8 \sin \lambda x) - \bar{Q},$$
$$\lambda = \sqrt[4]{\frac{\rho H \omega^2}{D_e}} \sqrt{\frac{1}{2\sqrt{\theta}} + \frac{\sqrt{1+\theta}}{\theta\sqrt{8}}}, \qquad æ = \sqrt[4]{\frac{\rho H \omega^2}{D_e}} \sqrt{\frac{1}{2\sqrt{\theta}} - \frac{\sqrt{1+\theta}}{\theta\sqrt{8}}}, \tag{2.2}$$
$$\bar{Q} = \frac{q_0}{\rho H \omega^2}, \qquad \theta = \sqrt{1 + \Big(\frac{D_\nu \omega}{D_e}\Big)^2}.$$

To construct the solution, we introduce three auxiliary variables ξ_i:

$$\xi_i = (\boldsymbol{r} - \boldsymbol{r}_i)\boldsymbol{n}_i \qquad (i = 1,\, 2,\, 3). \tag{2.3}$$

Here $\boldsymbol{r}$ is the radius-vector of an arbitrary point in the equilateral-triangle region Ω, $\boldsymbol{r}_i$ is the radius-vector of the triangle vertices, and $\boldsymbol{n}_i$ are the unit normal vectors to the triangle sides directed inward of Ω (Fig. 1). The variables ξ_i have the following geometrical meaning: the value of $\xi_i(x_0, y_0)$ calculated at the point $M(x_0, y_0)$ inside the equilateral-triangle region Ω is equal to the distance from this point to the ith side of the triangle.

The variables ξ_i have the following properties.

1. The equations of sides 1, 2, and 3 of the triangle (Fig. 1) have the forms $\xi_1 = 0$, $\xi_2 = 0$, and $\xi_3 = 0$, respectively.

2. The sum of the variables ξ_i is a constant. To show this, we use the equality

$$\boldsymbol{n}_1 + \boldsymbol{n}_2 + \boldsymbol{n}_3 = 0. \tag{2.4}$$

Using (2.3) and (2.4), we obtain $\xi_1 + \xi_2 + \xi_3 = \boldsymbol{r}(\boldsymbol{n}_1 + \boldsymbol{n}_2 + \boldsymbol{n}_3) - \boldsymbol{r}_1\boldsymbol{n}_1 - \boldsymbol{r}_2\boldsymbol{n}_2 - \boldsymbol{r}_3\boldsymbol{n}_3 = -\boldsymbol{r}_1\boldsymbol{n}_1 - \boldsymbol{r}_2\boldsymbol{n}_2 - \boldsymbol{r}_3\boldsymbol{n}_3$. Whence,

$$\xi_1 + \xi_2 + \xi_3 = \text{const}. \tag{2.5}$$

At the vertex of the triangle, we have

$$\xi_1 = 0, \qquad \xi_2 = 0, \qquad \xi_3 = h. \tag{2.6}$$

Using (2.6), for the sum of the variables ξ_i from (2.5) we arrive at the relation

$$\xi_1 + \xi_2 + \xi_3 = h, \tag{2.7}$$

where h is the height of the triangle. This equality implies that the sum of the distances from any point of the plane to the sides of the equilateral triangle is a constant equal to its height.

3. If the function $F(\xi_i)$ depends only on one geometrical variable ξ_i, the invariance of the Laplace operator relative to rotation of the coordinate system and translation of the coordinate origin implies the auxiliary differential relations

$$\nabla^2 F(\xi_i) = \frac{d^2 F(\xi_i)}{d\xi_i^2} = F'', \qquad \nabla^4 F(\xi_i) = \frac{d^4 F(\xi_i)}{d\xi_i^4} = F^{\rm IV}.$$

Substituting successively three variables ξ_i into the function $F(\xi_i)$, we obtain three equal functions $F(\xi_1)$, $F(\xi_2)$, and $F(\xi_3)$ of different variables. In addition to these functions, we introduce the functions

$$F(\xi_1+\xi_2), \qquad F(\xi_1+\xi_3), \qquad F(\xi_2+\xi_3), \tag{2.8}$$

which satisfy the differential equation (2.1). We now show this procedure only for $F(\xi_1+\xi_2)$.

With allowance for (2.7), we write the sum $\xi_1+\xi_2$ in the form $\xi_1+\xi_2 = h-\xi_3$, i.e., $F(\xi_1+\xi_2) = F(h-\xi_3)$. Since $F(\xi_3)$ from (2.2) satisfies the differential equation (2.1) for $x=\xi_3$, the function $F(h-\xi_3)$ satisfies Eq. (2.1) for $x = h-\xi_3$.

Using the principle of superposition, we express the solution of problem (2.1) in terms of three functions $F(\xi_i)$ and three functions (2.8):

$$U = F(\xi_1)+F(\xi_2)+F(\xi_3)-F(\xi_1+\xi_2)-F(\xi_2+\xi_3)-F(\xi_1+\xi_3)-\frac{q_0}{\rho H\omega^2-k}. \tag{2.9}$$

With allowance for (1.10), the boundary conditions take the form

$$U\Big|_\Gamma = F(0)-F(h)-\frac{q_0}{\rho H\omega^2} = a, \qquad \frac{\partial^2 U}{\partial n^2}\Big|_\Gamma = F''(0)-F''(h)=0,$$

$$V\Big|_\Gamma = \frac{\rho H\omega^2 D_e a + D_e q_0 - (D_e^2+D_\nu^2\omega^2)(F^{\rm IV}(0)-F^{\rm IV}(h))}{\rho H\omega^3 D_\nu} = b, \tag{2.10}$$

$$\frac{\partial^2 V}{\partial n^2}\Big|_\Gamma = F^{\rm VI}(0)-F^{\rm VI}(h)=0.$$

Substituting (2.2) into (2.9), we obtain the following solution for U:

$$\begin{gathered} U = C_1\varphi(\lambda,æ)+C_2\psi(\lambda,æ)+C_3\varphi(-\lambda,æ)+C_4\psi(-\lambda,æ) \\ +C_5\varphi(æ,\lambda)+C_6\psi(æ,\lambda)+C_7\varphi(-æ,\lambda)+C_8\psi(-æ,\lambda)-q_0/(\rho H\omega^2), \\ \varphi(\lambda,æ) = \mathrm{e}^{\lambda\xi_1}\cos æ\xi_1+\mathrm{e}^{\lambda\xi_2}\cos æ\xi_2+\mathrm{e}^{\lambda\xi_3}\cos æ\xi_3 \\ -\mathrm{e}^{\lambda(h-\xi_1)}\cos æ(h-\xi_1)-\mathrm{e}^{\lambda(h-\xi_2)}\cos æ(h-\xi_2)-\mathrm{e}^{\lambda(h-\xi_3)}\cos æ(h-\xi_3), \\ \psi(\lambda,æ) = \mathrm{e}^{\lambda\xi_1}\sin æ\xi_1+\mathrm{e}^{\lambda\xi_2}\sin æ\xi_2+\mathrm{e}^{\lambda\xi_3}\sin æ\xi_3 \\ -\mathrm{e}^{\lambda(h-\xi_1)}\sin \varkappa(h-\xi_1)-\mathrm{e}^{\lambda(h-\xi_2)}\sin \varkappa(h-\xi_2)-\mathrm{e}^{\lambda(h-\xi_3)}\sin \varkappa(h-\xi_3). \end{gathered} \tag{2.11}$$

Let us show that of eight functions $\varphi(\pm\lambda,æ)$, $\psi(\pm\lambda,æ)$, $\varphi(\pmæ,\lambda)$, and $\psi(\pmæ,\lambda)$ that enter (2.1), four functions $\varphi(-\lambda,æ)$, $\psi(-\lambda,æ)$, $\varphi(-æ,\lambda)$, and $\psi(-æ,\lambda)$ are expressed linearly in terms of the other four functions $\varphi(\lambda,æ)$, $\psi(\lambda,æ)$, $\varphi(æ,\lambda)$, and $\psi(æ,\lambda)$. This relation can be found provided a relation between the variables ξ_i (2.7) exists. With allowance for (2.7), the function $\varphi(-\lambda,æ)$ can be written in the form

$$\varphi(-\lambda,æ) = -\mathrm{e}^{-\lambda h}\,[\cos æh\varphi(\lambda,æ)+\sin æh\psi(\lambda,æ)]. \tag{2.12}$$

Similarly, we write the functions

$$\begin{gathered} \varphi(-æ,\lambda) = -\mathrm{e}^{-æh}[\cos\lambda h\varphi(æ,\lambda)+\sin\lambda h\psi(æ,\lambda)], \\ \psi(-æ,\lambda) = \mathrm{e}^{-æh}[\cos\lambda h\psi(æ,\lambda)-\sin\lambda h\varphi(æ,\lambda)], \\ \psi(-\lambda,æ) = \mathrm{e}^{-\lambda h}[\cos æh\psi(\lambda,æ)-\sin æh\varphi(\lambda,æ)]. \end{gathered} \tag{2.13}$$

By virtue of the linear relations (2.12) and (2.13), we can retain four linearly independent terms in the solution (2.11) instead of eight terms by setting, for example, $C_2=C_4=C_6=C_8=0$. In this case, after determination of C_1, C_3, C_5, and C_7 from the boundary conditions (2.10), the solution for U is expressed in terms of four functions $\varphi(\pm\lambda,æ)$ and $\varphi(\pmæ,\lambda)$:

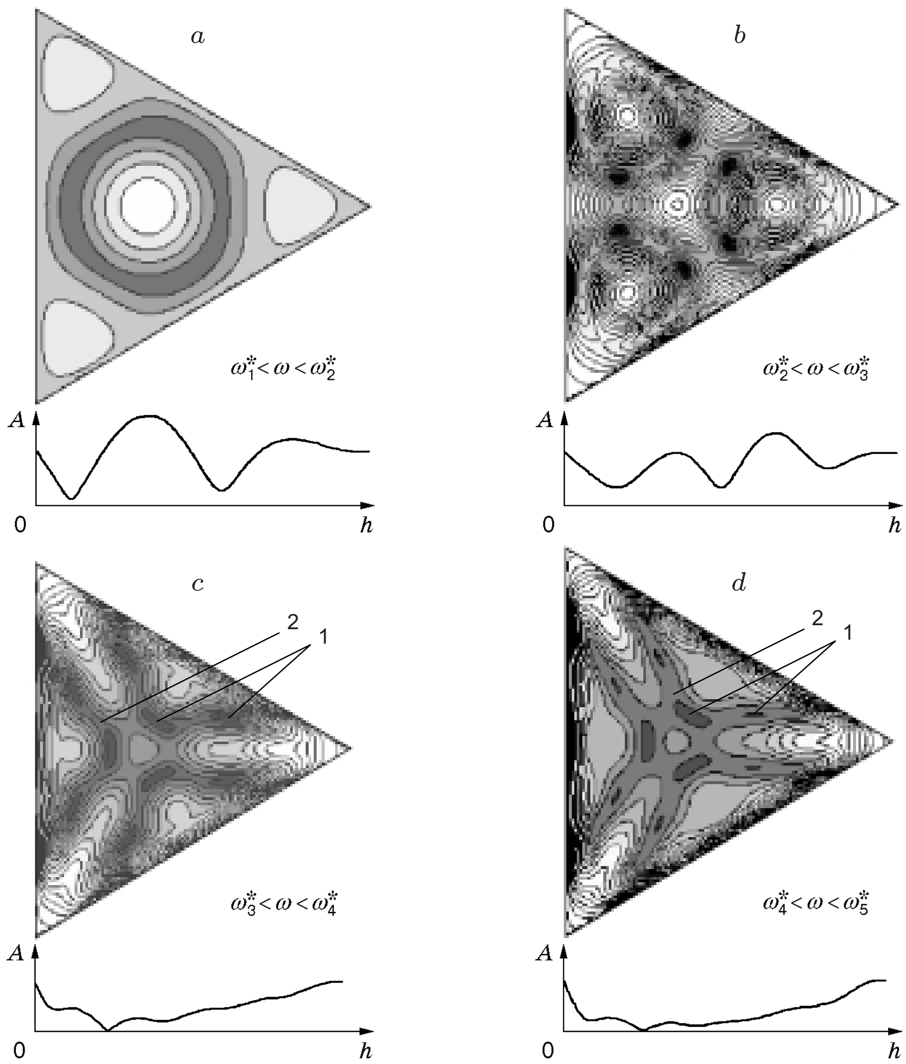

Fig. 2

$$U = C_1\varphi(\lambda, æ) + C_3\varphi(-\lambda, æ) + C_5\varphi(æ, \lambda) + C_7\varphi(-æ, \lambda) - q_0/(\rho H\omega^2), \tag{2.14}$$

where

$$C_1 = \frac{R(\lambda, æ)}{M(\lambda, æ)}, \quad C_3 = -\mathrm{e}^{2\lambda h}\,\frac{R(-\lambda, -æ)}{M(\lambda, æ)}, \quad C_5 = \frac{R(æ, -\lambda)}{M(æ, \lambda)}, \quad C_7 = -\mathrm{e}^{2æ h}\,\frac{R(-æ, \lambda)}{M(æ, \lambda)},$$

$$M(\lambda, æ) = 8\lambda æ(\lambda^2 - æ^2)\sin æ h(2\mathrm{e}^{\lambda h}\cos æ h - \mathrm{e}^{2\lambda h} - 1), \quad R(\lambda, æ) = N_1(\mathrm{e}^{\lambda h} - \cos æ h) - N_2 \sin æ h,$$

$$N_1 = \Big(a + \frac{q_0}{\rho H\omega^2}\Big)(\lambda^4 - 6\lambda^2 æ^2 + æ^4) - \frac{D_e q_0 + \rho H\omega^2(D_e a - D_\nu \omega b)}{D_e^2 + D_\nu^2\omega^2},$$

$$N_2 = 4\lambda æ(a + q_0/(\rho H\omega^2))(\lambda^2 - æ^2).$$

To find the solution for V, we first find $\nabla^4 U$:

$$\nabla^4 U = (\lambda^4 - 6\lambda^2 æ^2 + æ^4)(C_1\varphi(\lambda, æ) + C_3\varphi(-\lambda, æ) + C_5\varphi(æ, \lambda) + C_7\varphi(-æ, \lambda))$$

$$+ 4\lambda æ(æ^2 - \lambda^2)(C_1\psi(\lambda, æ) - C_3\psi(-\lambda, æ) - C_5\psi(æ, \lambda) + C_7\psi(-æ, \lambda)). \tag{2.15}$$

Substituting the known expressions (2.14) and (2.15) into (1.9), we obtain the solution for V:

$$V = \frac{(D_e \rho H \omega^2 - (D_e^2 + D_\nu^2 \omega^2)(\lambda^4 - 6\lambda^2 æ^2 + æ^4))G_1 - 4\lambda æ(æ^2 - \lambda^2)(D_e^2 + D_\nu^2\omega^2)G_2}{D_\nu \rho H \omega^3}. \quad (2.16)$$

Here

$$G_1 = C_1\varphi(\lambda, æ) + C_3\varphi(-\lambda, æ) + C_5\varphi(æ, \lambda) + C_7\varphi(-æ, \lambda),$$
$$G_2 = C_1\psi(\lambda, æ) - C_3\psi(-\lambda, æ) - C_5\psi(æ, \lambda) + C_7\psi(-æ, \lambda). \quad (2.17)$$

3. Properties of the Vibrations. In the particular case of an elastic plate ($\lambda_\nu = \mu_\nu = 0$), the resonance occurs for [4]

$$\omega_n^* = \frac{4\pi^2 n^2}{h^2}\sqrt{\frac{D}{\rho H}} \qquad (n = 1, 2, 3, \ldots).$$

Vibrations of viscoelastic systems are characterized by the absence of resonance, i.e., the amplitude of nonstationary vibrations of a plate tends to a certain maximum value, which depends on the viscous and elastic properties of the plate rather than to infinity. It is difficult to determine the amplitude of plate vibrations by analytical methods because of the cumbersome expressions for U and V. Therefore, we determine the amplitude numerically.

To study the behavior of a viscoelastic plate vibrating with different frequencies, it is important to find conditions under which the nodal zones with zero vertical displacements of the points on the middle surface of the plate appear. In our case, the vibration amplitude vanishes when $\sqrt{U^2 + V^2} = 0$.

Using the equality $U = 0$, from Eq. (2.14) we obtain

$$G_1 = q_0/(\rho H \omega^2), \quad (3.1)$$

where G_1 is given by (2.17). Substituting (3.1) into the solution (2.17) for $V = 0$, we obtain the following equation of nodal zones:

$$(D_e \rho H \omega^2 - (D_e^2 + D_\nu^2 \omega^2)(\lambda^4 - 6\lambda^2 æ^2 + æ^4))\,\frac{q_0}{\rho H \omega^2} - 4\lambda æ(æ^2 - \lambda^2)(D_e^2 + D_\nu^2 \omega^2)G_2 = 0.$$

In the absence of external loading, i.e., for $q_0 = 0$, this equation is simplified to $G_2 = 0$.

In practice, various structural members are usually made from materials having a different viscosity. To reveal the specific features of vibrations of these plates, we introduce a coefficient that characterizes the ratio between the viscous and elastic properties: $\chi = D_\nu / D_e$.

Figure 2 shows the level lines of the amplitude $A = \text{const}$ and the amplitude distribution along the height of an equilateral triangle ($h = 200$ cm) for a viscoelastic plate with $\chi = 0.0005$ sec for frequencies lying in the interval of the resonance frequencies of an elastic plate.

The behavior of the plate changes drastically, beginning from the frequency interval $\omega_3^* < \omega < \omega_4^*$. Nine oval nodal zones 1 with a close-to-zero amplitude of vibrations appear (Fig. 2c). It is noteworthy that in addition to these nodal zones, there are zones 2 with a small amplitude of vibrations (Fig. 2c) which form a complex figure with six oblong branches emanating from the plate center.

In summary, the following specific features of vibrations of a triangular plate are noteworthy. An analysis of the exact solution shows that the nodal zones that remain immovable during vibrations exist. Adjacent to the nodal zones are small-amplitude zones whose dimensions increase with vibration frequency. These zones are located near the center (Fig. 2d). With further increase in frequency, the small-amplitude zones become larger and occupy the entire central part of the plate. In this case, only regions adjacent to the plate boundary perform vibrations owing to the forced vibrations of the boundary.

REFERENCES

1. Y. Sing, M. Ahmad, and H. H. Hilton, "Dynamic responses of plates with viscoelastic free layer damping treatment," *Trans. ASME, J. Vibr., Acoust., Stress Reliability Design*, No. 118, 362–367 (1996).
2. Z. Yuanyuan and C. Changjun, "Stability analysis of viscoelastic rectangular plates," *Acta Mech. Solida Sinica*, No. 17, 275–282 (1996).
3. O. E. Boshnich, "Calculation of the vibrations of plates under various harmonic excitations with allowance for energy dissipation in a material," *Probl. Prochn.*, No. 4, 1170–129 (1997).
4. N. A. Chernyshov and A. D. Chernyshov, "Vibrations of an elastic triangular plate under the combined action of uniformly distributed transverse loading and uniform tension," *Izv. Inzh.-Tekhnol. Akad. Chuvash. Resp.*, No. 11, 87–95 (1998).
5. S. P. Timoshenko, *Vibration Problems in Engineering*, Van Nostrand, Toronto–New York–London (1955).